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# Sequential Decision Fusion for Abnormality Detection via Diffusive Molecular Communications

Sinem Solak, *Student Member, IEEE*, Mengüç Öner, *Member, IEEE*

**Abstract**—This paper considers the task of abnormality detection in a fluid medium, employing a molecular communications (MC) based network of nanoscale sensors. This task entails sensing, detection and reporting of abnormal changes in the environment that may characterize a disorder or an abnormal event. Such distributed detection (DD) problems are of paramount interest, especially in applications such as health monitoring, disease diagnosis, targeted drug delivery, environmental sensing and monitoring, contaminant detection and removal, and environmental remediation. This letter proposes, for the first time in the literature, to employ a sequential probability ratio test based approach to the decision fusion in diffusive MC based DD. The proposed approach leads to considerable gains in the average number of samples required for the decision compared to its fixed-sample size counterparts, resulting in a significant improvement in the average decision delay. In the investigated DD scenarios, we observe savings of up to 50% in the number of samples required for decision fusion.

**Keywords**—*Molecular Communications, distributed detection, nanoscale sensor networks.*

## I. INTRODUCTION

Molecular communications (MC) represents a promising biocompatible communications paradigm for nanoscale networks, mimicking the naturally evolved communication mechanisms between biological entities at this very small physical scale. Employing dedicated molecules as information carriers, diffusion based MC encodes information into some aspect of the released molecules, such as the release time, the number or the type of those molecules [1]. This paper focuses on the task of abnormality detection i.e. the detection and reporting of abnormal events that may characterize the presence of a disorder in a fluid environment, employing an MC based nanoscale sensor network [2]. Such distributed detection (DD) problems lie in the heart of the most highly anticipated applications of nanoscale networks, such as health monitoring, disease diagnosis, targeted drug delivery, environmental sensing and monitoring, contaminant and toxic agent detection, environmental remediation and many more. Depending on the application, the abnormalities of interest may be quite diverse in nature, e.g. abnormal changes in the concentration of a molecule in the medium, or abnormal changes in the properties of the medium itself, such as the pH value, temperature, viscosity, etc. [2].

Compared to the existing literature in wireless sensor networks based DD, research on MC based DD is still in a nascent state. The work presented in [3] and [4] investigates a case where the abnormality to be sensed is the change

in the concentration of a molecule. Both works model the MC channel between the sensors and the fusion centre (FC) as an additive white Gaussian noise multiple access channel with perfectly known received signal amplitudes. The work in [5] focuses on a similar type of abnormality as in [4], using a more realistic channel model based on the solution of the diffusion-reaction equations in an unbounded medium, and provides a sub-optimal decision fusion (DF) strategy. In [6], hard decisions are employed at the sensors, and a sub-optimal DF approach based on logic OR and AND operations. Both [5] and [6] assume perfectly orthogonal sensor-to-FC communication channels. This requires a receiver that can provide as many molecule receptor types as there are sensors, which may become impractical for applications involving a large number of sensors. The work in [2] employs an abstract and more general probabilistic sensing model, allowing soft decisions at the sensors, investigates two multiple access strategies to the channel for the sensor-FC communication and derives sub-optimal fixed sample-size tests for DF. The tests in [2] rely on approximations of the likelihood function of the received signal that require the signal sequence received at the FC to be independent and identically distributed (i.i.d), a condition that is approximately satisfied only if a sufficiently large number of molecule pulses is transmitted and a number of received samples at the beginning of the detection cycle are discarded until a steady-state is achieved at the FC. However, in a practical implementation, this requirement will result in additional decision delays beyond the length of the employed fixed-size observation window.

The DD strategies existing in the literature, summarized above, approach the task of DF by employing fixed sample size tests within the conventional Neyman-Pearson framework, with the detection probability for a specific false alarm rate for a given fixed number of channel observations  $N$  as the main performance criterion. However, one of the main characteristics of diffusive MC is the extremely slow signal propagation speed in the medium [7] and the highly dispersive nature of the channel, leading to long pulse intervals and long latency. Thus, DF schemes that require a large number of channel observations (i.e samples) at the FC, and/or rely on assumptions that may result in additional latency in practice, may lead to excessive decision delays. Consequently, performing a reliable DF with as few receive samples as possible is of paramount interest. This makes the use of sequential tests, which allow the use of variable observation window lengths in order to minimize the average number of observations required for decision, while retaining a prescribed detection performance, a promising and efficient alternative to fixed sample size based approaches for DF investigated in the literature.

In this work, we propose, for the first time in the literature, to

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Sinem Solak and Mengüç Öner are with the Dept. of Engineering and Design, University of Sussex, Brighton, UK. e-mail:s.solak@sussex.ac.uk, m.m.oner@sussex.ac.uk.

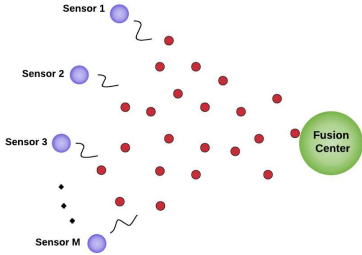


Fig. 1: The nanoscale sensor network structure under consideration.

employ a sequential probability ratio test (SPRT) based method for DF in diffusive MC based DD<sup>1</sup>. The proposed approach, which we refer to as the sequential average probability ratio test (SAPRT), is based on the SPRT proposed by Wald in [9]. We have chosen the fixed sample size test in [2] as a benchmark for performance comparison due to its generality, flexibility and practical relevance. Our results show that the proposed sequential approach leads to significant savings in the average number of samples required for DF, and, consequently, to a considerable reduction in the decision delay, while achieving the same average detection performance without relying on an i.i.d. assumption that may lead to additional decision delays in practice.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a centralized sensor network, with  $M$  identical sensors transmitting their soft decisions to a FC via diffusion based MC as illustrated in Fig.1. With the hypotheses  $\mathcal{H}_0$  and  $\mathcal{H}_1$  representing the absence and the presence of the abnormality of interest, respectively, the task of the FC is to map the received signals from the sensors via the MC channel to a decision  $\hat{\mathcal{H}} \in \{\mathcal{H}_0, \mathcal{H}_1\}$ . In order to provide a fair comparison with the fixed sample size strategy in [2] chosen as a benchmark, we employ the same abstract sensing model and the same communication model between the sensors and the FC, which will be described in the rest of this section.

1) *The Sensing Model:* Due to the broad spectrum of potential applications envisioned for nanoscale sensor networks, a wide range of abnormalities representing diverse physical or biochemical phenomena may become of practical interest, which require different sensing mechanisms. While some of the existing works have been focusing on a specific type of abnormality and sensing model (e.g. [4], [5]), others, such as [2] and [6], have employed abstract sensing models in order to achieve more general results. In this work, we chose to focus on the latter approach for the sake of generality. Each of the  $M$  sensors is assumed to measure one or more sensing variables and generate a quantized soft output between 0 and 1 representing its sensing decision, i.e. the  $m$ 'th sensor's output  $X_m \in \mathcal{S} = \{0, 1/(L-1), 2/(L-1), \dots, 1\}$  where  $L$  is the number of quantization levels and  $m \in \{1, 2, \dots, M\}$ . Clearly, this model can accommodate both hard decisions (for  $L = 2$ ) and soft decisions (for  $L > 2$ ) at the sensors.

<sup>1</sup>Note that an SPRT based approach is employed in a point-to point MC system for increasing the robustness of data demodulation in [8], i.e. in a distinctly different context with a different purpose

The uncertainties in the sensor outputs due to measurement imperfections associated with the sensing mechanism, e.g. due to sensor noise are accounted for by modeling the sensor outputs  $X_m$  as random variables with a conditional probability mass function (pmf)  $q_i(x_m)$ :

$$q_i(x_m) = P(X_m = x_m | \mathcal{H}_i), \text{ for } i = 0, 1. \quad (1)$$

2) *The Reporting Model:* As in [2], we assume perfect point transmitters at the sensors, a perfectly absorbing spherical receiver model at the FC, and an unbounded medium for diffusion. It has been shown in [10] that for a molecule released by a point transmitter located at a distance of  $r_1$  from the center of a perfectly absorbing spherical receiver of radius  $r_2$ , the probability of hitting the surface of the receiver within a time interval  $[kT, (k+1)T]$  seconds after release is given as:

$$p_k = \begin{cases} \frac{r_2}{r_1} \operatorname{erfc}\left(\frac{r_1 - r_2}{\sqrt{4DT}}\right), & \text{for } k = 0 \\ \frac{r_2}{r_1} \left( \operatorname{erfc}\left(\frac{r_1 - r_2}{\sqrt{4(k+1)DT}}\right) - \operatorname{erfc}\left(\frac{r_1 - r_2}{\sqrt{4kDT}}\right) \right), & \text{for } k \geq 1, \end{cases} \quad (2)$$

where  $D$  is the diffusion coefficient of the information carrying molecule in the medium and  $\operatorname{erfc}(\cdot)$  is the complementary error function. Each of the  $M$  sensors transmits its output to the FC starting at the time instant  $t = 0$  by releasing  $N$  consecutive pulses of  $X_m A$  information carrying molecules, each  $T$  seconds apart, where  $A$  represents the maximum number of molecules available for each pulse. Hence, the sensor output  $X_m$  modulates the amplitude of the transmit pulse train of the corresponding sensor. In this work we focus on a case, where a single molecule type is employed for communication by all sensors, allowing the use of a simpler receiver in practice. The sensors are assumed to be equidistant to the FC with statistically independent sensing measurements. The receive signal at the FC,  $Y_n$ , is defined as the random sequence representing the number of molecules absorbed by the FC within the time slot  $[(n-1)T, nT]$ . In such a case, for a given realisation of the sensor outputs  $X_m = x_m$ ,  $m = 1, \dots, M$ ,  $Y_n$  can be modeled as an independent Poisson distributed random sequence with a time-varying mean:

$$Y_n | X_m = x_m \sim \operatorname{Pois}\left(J + \sum_{m=1}^M \sum_{k=0}^n p_k x_m A\right), \quad (3)$$

where  $J$  is the expected value of the received Poisson-distributed additive noise molecules [10]. Note that the independence of the sequence  $Y_n$  is easily verified, assuming regular brownian diffusion, a large number of molecules, and a perfectly absorbing receiver which irreversibly removes all the molecules crossing across its surface. The task of the FC is to perform the decision fusion (DF), i.e. to decide for the hypothesis  $\mathcal{H}_0$  or  $\mathcal{H}_1$  by observing  $y_n$ , a realisation of the sequence  $Y_n$ , which will be investigated in the next section.

## III. SEQUENTIAL DECISION FUSION

As discussed above, existing DF strategies in the literature are based on fixed sample size tests within the Neyman-Pearson framework, that have to wait to reach a decision until a fixed number of samples are received, resulting a fixed decision delay. In this section, we propose a sequential test for DF,

which, on average, requires a much lower number of channel observations for DF for the same decision performance, leading to a considerable improvement in the average decision delay.

1) *The Average Likelihood Function of the observations:* Using (3), the conditional pmf of an observation is given as:

$$P(Y_n=y_n|X=x, \mathcal{H}_i) = \frac{e^{-J-x\sum_{k=0}^n p_k A} (J+x\sum_{k=0}^n p_k A)^{y_n}}{y_n!}, \quad (4)$$

where  $X = \sum_{m=1}^M X_m$  is the sum of the sensor outputs and  $x \in \mathcal{X} = \{0, 1/(L-1), 2/(L-1), \dots, M\}$  is a realization of  $X$ .

When deriving its DF rules, the benchmark fixed sample size method in [2] approximates the term  $\sum_{k=0}^n p_k A$  in (4) with its limit as  $n \rightarrow \infty$ , with the assumption that a large number of pulses are transmitted, and a number of the samples received at the FC at the beginning of an observation window are discarded until a steady state is achieved. While these assumptions make  $Y_n$  stationary and i.i.d., simplifying the derivation and the analysis of the DF rule, they imply the presence of an additional decision delay beyond the fixed observation window length parameter  $N$  that does not account for the discarded samples, and also an inefficient utilisation of the molecules available for transmission. Since our primary focus is the decision delay, we chose to employ the exact non stationary conditional distribution in the derivation of the average likelihood function of the observations.

Let  $\mathbf{Y}^{(l)}$  represent the random vector containing  $l$  consecutive samples of  $Y_n$ , i.e.  $\mathbf{Y}^{(l)} = [Y_1, Y_2, \dots, Y_l]^T$ , corresponding to an observation window length of  $(l-1)T$  seconds. Using (4) and the independence of the sequence  $Y_n$ , the conditional pmf of  $\mathbf{Y}^{(l)}$  for a given realization of  $X = x$  and hypothesis  $\mathcal{H}_i$  can be expressed as:

$$P(\mathbf{Y}^{(l)} = \mathbf{y}^{(l)} | X=x, \mathcal{H}_i) = \exp(-Jl - \sum_{n=1}^l x B_n) \prod_{n=1}^l \frac{(J+x B_n)^{y_n}}{y_n!}, \quad (5)$$

with  $\mathbf{y}^{(l)} = [y_1, y_2, \dots, y_l]^T$  a realization of  $\mathbf{Y}^{(l)}$  and  $B_n = \sum_{k=0}^n p_k A$ . Clearly, during detection, it is not possible for the FC to have any a-priori information on the current realization of  $X$ . Thus,  $X$  is treated as a nuisance parameter by averaging (7) over its conditional pmf.

$$P(\mathbf{Y}^{(l)} = \mathbf{y}^{(l)} | \mathcal{H}_i) = \sum_{x \in \mathcal{X}} P(\mathbf{Y}^{(l)} = \mathbf{y}^{(l)} | X=x) P(X=x | \mathcal{H}_i). \quad (6)$$

Hence, the average likelihood function (over  $X$ ) of a received signal vector  $\mathbf{y}^{(l)}$  of length  $l$  is calculated as:

$$\begin{aligned} P(\mathbf{Y}^{(l)} = \mathbf{y}^{(l)} | \mathcal{H}_i) &= \sum_{x \in \mathcal{X}} \left( Q_i^{(M)}(x) \exp(-Jl - \sum_{n=1}^l x B_n) \right. \\ &\quad \times \left. \prod_{n=1}^l \frac{(J+x B_n)^{y_n}}{y_n!} \right), \end{aligned} \quad (7)$$

where  $Q_i^{(M)}(x)$  is the conditional pmf of  $X$ , the aggregated sensor output and can be calculated as:

$$Q_i^{(M)}(x) = P(X=x | \mathcal{H}_i) = \underbrace{q_i(x) * q_i(x) * \dots * q_i(x)}_{M-1 \text{ times}}, \quad (8)$$

i.e. by convolving  $q_i(x)$  in (1)  $M-1$  times with itself, due to the statistical independence of the individual sensor outputs  $X_m$ . Hence, the average log-likelihood ratio (ALLR) of  $\mathbf{y}^{(l)}$  for this binary detection problem, is given as

$$\tilde{\Lambda}_Y(l) = \log \left\{ \frac{P(\mathbf{Y}^{(l)} = \mathbf{y}^{(l)} | \mathcal{H}_1)}{P(\mathbf{Y}^{(l)} = \mathbf{y}^{(l)} | \mathcal{H}_0)} \right\} \quad (9)$$

with  $P(\mathbf{Y}^{(l)} = \mathbf{y}^{(l)} | \mathcal{H}_i)$  calculated as in (7).

2) *The SAPRT for sequential Decision Fusion:* Sequential tests are equipped with a stopping rule that decides, at each time epoch, whether to wait and collect one more sample or to terminate and chose one of the hypotheses, and a decision rule that decides for  $\mathcal{H}_0$  or  $\mathcal{H}_1$  based on the samples available up to the stopping time. Defining  $\Lambda(l)$  as the log-likelihood ratio (LLR) of a vector of  $l$  samples of the received signal,  $l = 1, 2, \dots$ , the SPRT, proposed by Wald in [9] is given as:

$$\tau = \inf(l > 0 : \Lambda(l) \notin (S, U)) \quad (10)$$

$$\Gamma_\tau \triangleq \begin{cases} \text{Chose } \mathcal{H}_0, & \text{if } \Lambda(\tau) \leq S, \\ \text{Chose } \mathcal{H}_1, & \text{if } \Lambda(\tau) \geq U, \end{cases} \quad (11)$$

where  $\tau$  is the stopping time of the test and  $\Gamma_\tau$  is the decision rule. In other words, at each time epoch  $l$ , the running log-likelihood ratio  $\Lambda(l)$  is compared with a lower and upper threshold  $S$  and  $U$ ,  $S \leq U$ . If  $\Lambda(l)$  remains within the interval  $(S, U)$ , the test decides to collect one more sample, updates the LLR and repeats the procedure for  $l+1$ . The stopping time  $\tau$  of the SPRT is defined as the time instant where the value of the LLR exits the interval  $(S, U)$  for the first time. At the stopping time  $l = \tau$  the test terminates, and decides for  $\mathcal{H}_0$  if  $\Lambda(\tau) \leq S$ , and for  $\mathcal{H}_1$  if  $\Lambda(\tau) \geq U$ . Clearly, the stopping time  $\tau$  is a random variable, since its value depends on the random input sequence, and its expected value  $E\{\tau\}$  characterizes the average sample number (ASN) of the test. Wald has shown that, for a binary hypothesis test with i.i.d observations, the SPRT minimizes the ASN over both hypotheses for a given false alarm probability  $P_f = P\{\hat{\mathcal{H}} = \mathcal{H}_1 | \mathcal{H}_0\}$  and probability of detection  $P_d = P\{\hat{\mathcal{H}} = \mathcal{H}_1 | \mathcal{H}_1\}$  pair [11].

For the DF scenario considered in this work, we propose to use the ALLR  $\tilde{\Lambda}_Y(l)$  of the observation sequence given in eq. (9) in the SPRT described in eqs. (10) and (11), resulting in a test which we will refer to as the SAPRT, where, in contrast to the usual LLR based SPRT model, the running ALLR function  $\tilde{\Lambda}_Y(l)$  of the observations cannot be expressed as a running sum of the log likelihood ratios of the individual samples due to the averaging operation over the pmf of the nuisance parameter  $X$  performed in (6), despite the fact that  $Y_n$  is an independent (albeit not i.i.d.) sequence.

The performance of a SPRT is characterized by its operating characteristics (OC) and the ASN. While Wald's approximation [9] provides expressions for bounds of  $S$  and  $U$  in terms of the  $P_f$  and  $P_d$ , which can be used to find approximate values for  $S$  and  $U$ , exact calculations of these parameters, and analytical derivation of the OC and the ASN is only possible for some special cases. [12] shows that the OC and ASN functions obey the Fredholm integral equations of the second kind for the i.i.d. case which may be evaluated numerically

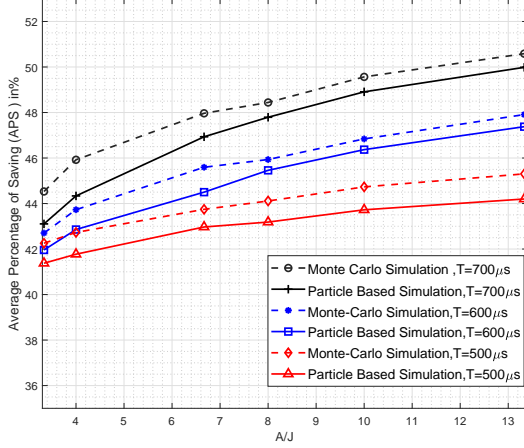


Fig. 2: APS vs.  $A/J$  for the SAPRT based DF, compared to the fixed sample size test in [2] for  $M = 4$ ,  $P_d = 0.999$ ,  $P_f = 0.001$ ,  $T = 500, 600$  and  $700\mu s$ ,  $(c_0, c_1) = (6.5, 7.5)$ ,  $N = 10$ .

(and for some simple cases, analytically). For the case of independent but not identically distributed (i.e. non-stationary) observations, [13] demonstrates a methodology for numerically approximating the OC and the ASN for some simple input distributions, which relies on recursively solving the governing integral equations. For the case of the SAPRT in the DF scenario considered in this work, the observation sequence  $Y_n$  is not i.i.d., and neither can the average LLR  $\hat{\Lambda}_Y(l)$  be expressed as a running sum of independent random variables as the LLR functions in [9], [12] and [13] as discussed above. Thus, an analytical derivation of the OC and ASN functions for this case remains mathematically intractable. However, the simulation results provided in the next section show that the proposed SAPRT based DF provides significant improvements in the average number of samples required for detection, and, subsequently, considerably less average decision delay compared to its fixed-sample-size counterpart in [2].

#### IV. RESULTS

In this section, the performance of the proposed SAPRT based sequential DF strategy is evaluated via both monte carlo simulations using the ideal signal model in (3) and (4), and particle based simulations, where the particle based molecular communication simulator AcCoRD [14] has been employed in the signal generation, using a diffusion timestep of  $5\mu s$ . In all cases, the diffusion coefficient  $D$  is chosen as  $D = 500\mu m^2 s^{-1}$ , in the same order of magnitude as the diffusion coefficients of small to medium sized biomolecules in blood plasma [15]. The size of the FC,  $r_2 = 3\mu m$  has been chosen within the same range as a bacterial cell and the sensor-FC distance is  $r_1 = 6\mu m$ . The number of quantization levels for the soft decisions at the sensors is chosen as  $L = 4$  and the sensing imperfections are modeled with the following conditional pmfs;

$$q_0(x_m) = \frac{\exp(-c_0 x_m)}{\sum_{x \in \mathcal{S}} \exp(-c_0 x)}, \quad (12)$$

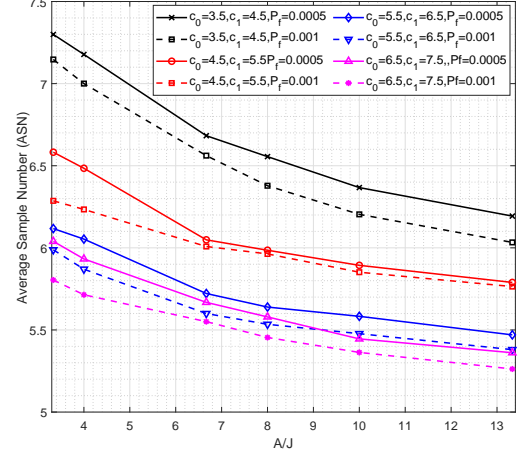


Fig. 3: The ASN of the SAPRT based DF for  $M = 4$ ,  $P_d = 0.999$  with  $P_f = 0.001$  and  $0.0005$  respectively, with  $M = 4$ ,  $T = 600\mu s$  under varying levels of sensor uncertainty. Only particle based simulations are considered.

$$q_1(x_m) = \frac{\exp(c_1 x_m)}{\sum_{x \in \mathcal{S}} \exp(c_1 x)}. \quad (13)$$

The coefficients  $c_0$  and  $c_1$  determine the sensing uncertainty of the individual sensors. The higher the values of these coefficients, the less uncertainty exists in the sensing decisions under each hypothesis and vice versa, allowing to model a wide range of sensor conditions. We use the ratio  $A/J$ , i.e. the ratio of the maximum number of molecules available for a pulse to the expected number of noise molecules received at each time slot, as our measure of the signal to noise ratio. The efficiency of the proposed sequential test compared to the benchmark fixed sample size test in terms of decision delay is measured by the quantity Average Percentage Saving (APS) [16], which quantifies the saving in the average number of samples required for the decision achieved by the proposed test relative to the benchmark fixed sample size test in [2], i.e:

$$APS = \frac{N - ASN}{N} \times 100\%, \quad (14)$$

where  $N$  is the sample size that the benchmark test requires to reach a decision for a given  $(P_f, P_d)$  pair, and the ASN is the average number of samples required by the proposed SAPRT to achieve the same detection performance, under the same conditions. Fig.2 displays the APS of the SAPRT compared to the fixed sample size test in [2] chosen as a benchmark vs.  $A/J$ , both for the monte-carlo simulations based on the ideal model, and for particle based simulations. The simulations have been performed with  $M = 4$  sensors,  $P_d = 0.999$ ,  $P_f = 0.001$ , and  $T = 500, 600$  and  $700\mu s$ ,  $N = 10$  and  $(c_0, c_1) = (6.5, 7.5)$ . Clearly, the APS results for the ideal monte carlo and particle-based simulations agree well, ca. within one percentage point across the board, where the particle-based case slightly underperforms due to the likelihood mismatch between the ideal model and the signal generated by the particle based simulations, caused by the slightly lower signal mean achieved in the latter. Compared



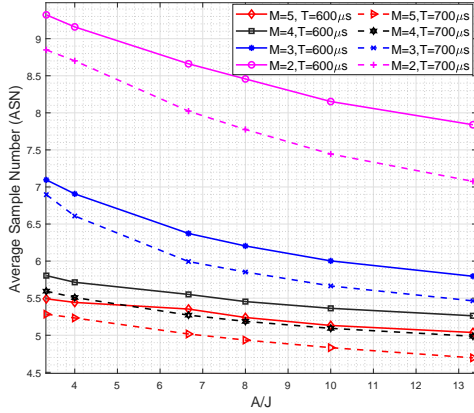


Fig. 4: The ASN of the proposed SAPRT based DF for  $P_d = 0.999$  with  $P_f = 0.001$ ,  $T = 600\mu s$  and  $700\mu s$ ,  $(c_0, c_1) = (6.5, 7.5)$ ,  $M = 2, 3, 4$  and  $5$ . Only particle based simulations are considered.

to the fixed sample size benchmark, the SAPRT achieves a considerable reduction in the average number of samples required for detection, leading to a significant decrease in the average decision delay. The results show that, as expected, the APS increases both with the ratio  $A/J$  [16] and  $T$ .

In the rest of the results, particle based diffusion simulations have been employed exclusively. Fig.3 illustrates the effect of the sensor uncertainties on the performance of the SAPRT in terms of the average sample size ASN (in samples) required to achieve  $P_d = 0.999$  with  $P_f = 0.001$  and  $0.0005$  respectively, for the same network with  $M = 4$ ,  $T = 600\mu s$ . Here, four  $(c_0, c_1)$  pairs are chosen to model different sensing conditions, from excellent to moderate, in that order:  $(c_0, c_1) = (6.5, 7.5)$ ,  $(5.5, 6.5)$ ,  $(4.5, 5.5)$  and  $(3.5, 4.5)$ . As expected, the SAPRT requires more samples to decide in order to achieve the required performance, as the sensor uncertainty increases. Furthermore, increasing  $P_f$  leads to a decrease in the ASN in all cases, which is also within expectations (see [16] for details). Finally, Figure 4 displays the effect of the number of sensors on the ASN for  $T = 600\mu s$ , and  $700\mu s$ ,  $P_d = 0.999$  with  $P_f = 0.001$  and  $M = 2, 3, 4, 5$  respectively, where the detection performance increases (i.e. the ASN decreases) with increasing number of sensors  $M$ .

## V. CONCLUSION

This work considers, for the first time in the literature, the use of a SPRT based test for the DF in a DD problem employing an MC based nanoscale sensor network. The results show that the proposed SAPRT achieves considerable savings in the number of samples required for decision compared to an existing fixed-sample-size Neyman-Pearson benchmark test based on a maximum likelihood approach, while attaining the same detection performance. Furthermore, the proposed method does not rely on simplifying approximations that, in practice, may lead to additional decision delays. This significant reduction in the decision delay makes the proposed strategy especially suitable for MC based DD problems, where the decision delay may become a major performance parameter. The proposed methodology is general, in the sense that it can

be employed under any type of diffusion dynamics (i.e, flow, reactions, anomalous diffusion, etc.), as long as the complete likelihood function of the receive signal is available at the FC, which, however, requires the knowledge of the all relevant system parameters. Note that, for practical cases, where some of the system parameters are unknown, and have to be estimated, the performance of the proposed DF methodology provides an upper performance bound. Our future research will include the investigation of non-parametric and distribution-free sequential DF approaches which are known to be exceptionally robust to parameter uncertainties and model mismatches, that are expected to be encountered in practical implementations within dynamic and highly complex biological environments.

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